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REFERENCES

[1] A. Korpel and P. P. Banerjee, "A heuristic guide to nonlinear dispersive wave equations and soliton-type solutions," *Proc. IEEE*, vol. 72, pp. 1109–1130, 1984.
[2] R. Y. Chiao, E. Garmire, and C. H. Townes, "Self-trapping of optical beams," *Phys. Rev. Lett.*, vol. 13, pp. 479–482, 1964.
[3] A. W. Snyder, D. J. Mitchell, L. Poladian, and F. Ladouceur, "Self-induced optical fibers: spatial solitary waves," *Opt. Lett.*, vol. 16, pp. 21–23, 1991.
[4] P. J. B. Clarricoats and B. C. Taylor, "Evanescent and propagating modes of dielectric-loaded circular waveguide," *Proc. Inst. Elec. Eng.*, vol. 111, pp. 1951–1956, 1964.
[5] S. B. Rayevskiy, "Some properties of complex waves in a double-layer circular, shielded waveguides," *Radio Eng. Electron Phys.*, vol. 21, pp. 36–39, 1976.
[6] U. Crombach, "Complex waves on shielded lossless rectangular dielectric image guide," *Electron. Lett.*, vol. 19, pp. 557–558, 1983.
[7] A. S. Omar and K. F. Schünemann, "The effect of complex modes at finline discontinuities," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 1508–1514, 1986.
[8] W. X. Huang and T. Itoh, "Complex modes in lossless shielded microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 163–165, 1988; C. J. Railton and T. Rozzi, "Complex modes in boxed microstrip," *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 865–874, 1988.
[9] A. A. Abrikosov, *Fundamentals of the Theory of Metals.* Amsterdam: North-Holland, 1988.
[10] A. A. Abrikosov, L. P. Gorkov, and I. E. Dzyaloshinski, *Methods of Quantum Field Theory in Statistical Physics.* Englewood Cliffs, N.J.: Prentice-Hall, 1963, p. 37.

On the Computation of Complex Modes in Lossless Shielded Asymmetric Coplanar Waveguides

Khwaja M. Rahman and Cam Nguyen

***Abstract*—We compute complex modes in lossless shielded asymmetric coplanar waveguides (CPW's) using the spectral domain technique. The slot asymmetry is found to significantly affect the existence of the complex modes. These modes are found to exist at low microwave frequencies even when using materials with a low permittivity. We found that waveguide modes degenerate into complex modes more frequently than CPW (π) and slotline (c) modes. When the structures are highly asymmetrical and when the dielectric substrates are thick or have a high permittivity, the degeneration of lower-order c-modes into complex modes is detected. Other forms of mode conversion, where a waveguide mode is converted to a c-mode, are also observed, especially in highly asymmetric structures and when using dielectric materials of a high permittivity or of a large thickness. Numerical convergence of the complex modes' propagation constants is also examined.**

I. Introduction

Since its discovery in 1969 by C. P. Wen [1], coplanar waveguide (CPW) has been used widely for microwave integrated circuits (MIC's) and monolithic microwave integrated circuits (MMIC's) [2]. Among CPW's, the asymmetrical version is very attractive since it can provide additional circuit design flexibility and improved characteristic impedance range. Various dynamic [3] and quasi-static [4] analyses have been performed for asymmetric CPW's. However, the analysis of complex modes in asymmetrical CPW's has not yet been addressed. As will be seen, the existence of complex modes is highly pronounced in asymmetrical CPW's. They have been found even at low microwave frequencies and in low permittivity substrates. A thorough knowledge of these complex modes is thus very important for the accurate design of MIC's and MMIC's using asymmetric CPW's in both low and high microwave regions.

In the past several years, complex modes in lossless waveguiding structures have been studied by a number of researchers. The presence of complex modes in lossless waveguiding structures was first predicted for a circular dielectric-loaded waveguide [5]. Later, theoretical as well as experimental investigations were made on the circular dielectric-loaded waveguide to confirm the existence of these modes [6], [7]. Complex modes were also reported for lossless finlines [8] and shielded microstrip lines [9], [10].

We present in this paper an extensive investigation of complex modes in lossless shielded three-layer asymmetric CPW's using the spectral domain approach (SDA) [11]. The effects of slot asymmetry, and dielectric constant and thickness of dielectric materials on the possible existence of complex modes, are described. Special attention is given to the numerical convergence of the calculated complex modes' propagation constants. The developed analysis has been applied to a symmetric CPW, and generated numerical results of the propagation constants of several real modes agree well with previously published data [12]. It should be noted here that our considered three-layer asymmetric CPW's are general in that they are applicable to both open and shielded CPW's, both symmetry and asymmetry in slots and ground planes, with and without a back-side conductor, with and without dielectric overlay, and with finite- and infinite-extent substrates. They can elude the energy leakage or increase the single-mode operating range with properly chosen dielectric substrates [13]. It is therefore expected that these asymmetrical CPW structures can be exploited to achieve MIC's and MMIC's with enhanced performance and smaller size.

II. Numerical Results and Discussions

Complex modes in a lossless shielded three-layer asymmetric CPW's with assumed infinitesimally thin metallization (Fig. 1) are investigated using the SDA. Applying the SDA produces a system of homogeneous linear equations. By setting the determinant of the coefficient matrix of the resultant equations to zero, we can solve for the propagation constants, γ, of all of the eigenmodes. The values of γ will be searched in the complex plane, owing to the fact that it is complex for complex modes. Due to the asymmetry in the structure, both the CPW (π) and slotline (c) modes will be excited along with the waveguide modes, leading to the possible existence of complex π-, c-, and waveguide modes. These complex modes appear in pairs and are formed when two evanescent modes degenerate into a pair of modes having $\gamma = \alpha \pm j\beta$, and which propagate in the $\pm$ directions with a phase constant β and attenuation constant α. These respective waves attenuate and grow exponentially with α as they propagate, leading to no corresponding transmitted power. Their existence is noticed when the root of the eigenvalue equation is complex, in spite of the lossless transmission line assumption.

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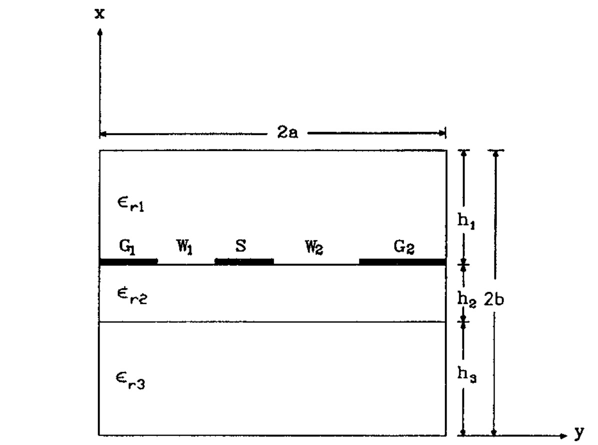
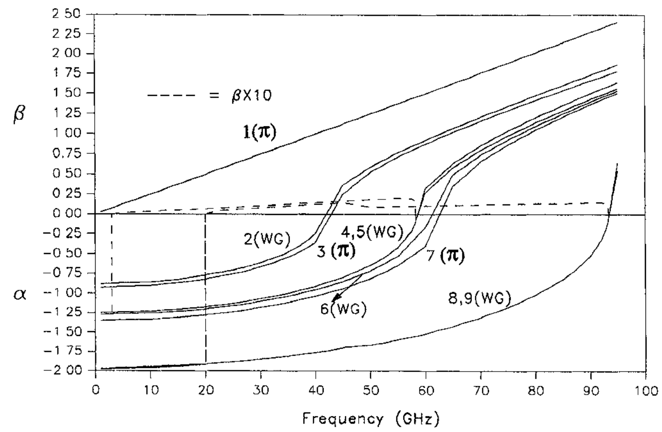


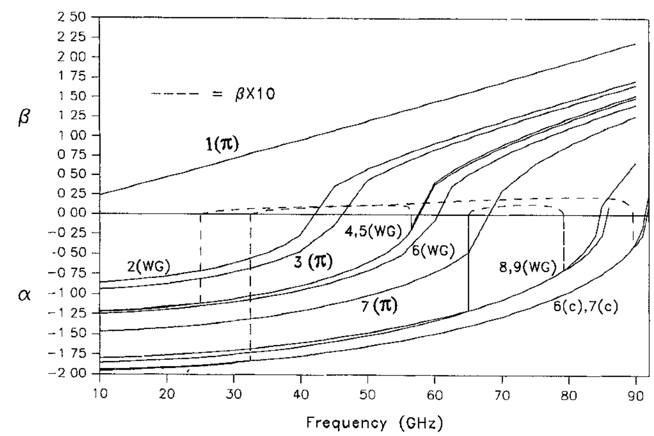
Fig. 1. Asymmetric CPW cross section.

TABLE I
COMPARISON OF THE EFFECTIVE DIELECTRIC CONSTANTS OF THE FIRST THREE MODES IN A SYMMETRIC CPW BETWEEN OUR ANALYSIS AND [12]. $\varepsilon_{r1} = \varepsilon_{r3} = 1$, $\varepsilon_{r2} = 9.6$, $h_1 = h_3 = 3$ mm, $h_2 = 1$ mm, $a = 7.5$ mm, $S = 2$ mm, $W_1 = W_2 = 1$ mm, $G_1 = G_2 = 5.5$ mm, $f = 30$ GHz. $P = Q = L = M = 3$

ϵ_{eff}					
Ref[12]			Ours		
Fundamental	First	Second	Fundamental	First	Second
5.819	4.371	3.033	5.8175	4.3646	3.0242



(a)



(b)

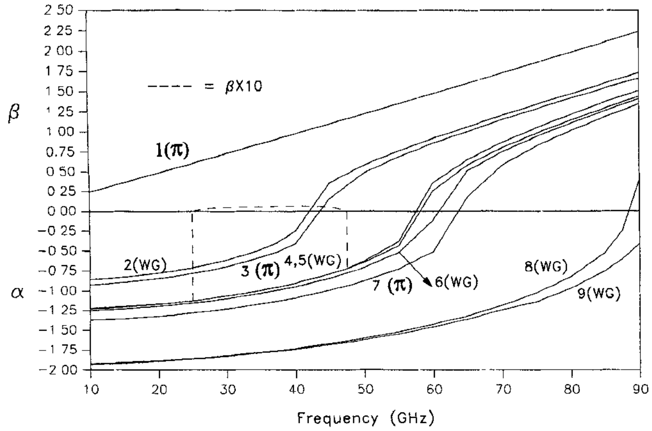
Fig. 2. Dispersion characteristics of evanescent and complex modes in a shielded asymmetric CPW as a function of slot asymmetry. $S = 0.762$ mm, $W_2 = 0.508$ mm, $G_2 = 0.889$ mm, $h_1 = h_3 = 3.429$ mm, $h_2 = 0.254$ mm, $\varepsilon_{r1} = \varepsilon_{r3} = 1$, $\varepsilon_{r2} = 2.2$ mm. (a) $W_1 = 0.127$ mm. (b) $W_1 = 1.27$ mm.

To verify our method, we calculated the propagation constants of the fundamental as well as the first and second higher-order real modes for a symmetrical CPW, and compared them with the existing data [12] in Table I. Good agreement was obtained, as can be seen in the table.

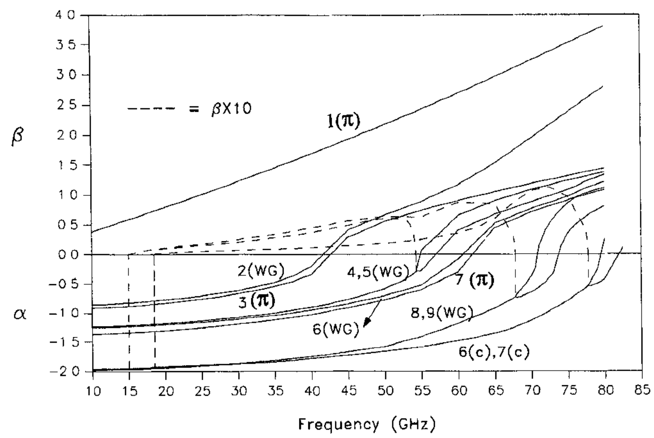
In all of the figures to be presented, the π- and waveguide modes are numbered in sequence: The fundamental mode is designated as 1; the first higher-order mode as 2; and so on. The c-modes are numbered separately. Furthermore, p-modes, c-modes, and waveguide modes are labeled "π," "c," and "WG" in the figures, respectively. In order to make the figures less crowded, only propagation constants of the c-modes taking part in the degeneration process are plotted. The phase constants, β, of the complex modes are identified by dashed curves and are amplified 10 times to make the variations more visible. Up to 18 modes were calculated in all of the considered structures. The enclosure used in these structures is a WR-28 rectangular waveguide having $2a = 3.556$ mm and $2b = 7.112$ mm.

Fig. 2(a) and (b) shows the effects of the slot asymmetry as well as frequency on the propagation constants of the fundamental and several higher-order modes, including the complex modes. In these plots, the slot width W_2, line width S, and ground-plane width G_2 are kept constant, while the slot width W_1 is varied. Various values of W_1 from 0.127–1.27 mm have been used. However, only the results corresponding to $W_1 = 0.127$ and 1.27 mm are presented here to illustrate the possible existence of the complex modes. In Fig. 2(a), where $W_1 = 0.127$ mm, only waveguide modes are found to degenerate into complex modes. Waveguide modes 4 and 5 degenerate into a complex conjugate pair at as low as 3 GHz, and remain so until 58.2 GHz, when they change back into evanescent modes. Modes 8 and 9, which have also been identified as waveguide modes, degenerate into a complex conjugate pair at 20 GHz and change back to evanescent modes at about 93.3 GHz. When W_1 is increased to 1.27 mm, corresponding to the dispersion curves drawn in Fig. 2(b), the complex modes corresponding to waveguide modes 4 and 5 are found to exist between 25 and 56.5 GHz. Waveguide modes 8 and 9 now convert into c-modes and, in the range of 65–79.25 GHz, they degenerate into complex modes. Moreover, a new pair of modes, c-modes 6 and 7, are found to degenerate into a complex mode pair within the frequency range of 32.5–89.6 GHz. The foregoing results indicate that waveguide modes 4 and 5 are always found to degenerate into a complex mode pair in all of the structures studied so far. On the other hand, waveguide modes 8 and 9 are generally found to convert into a complex mode pair for narrow slot widths (e.g., $W_1 = 0.127$ mm), and the c-modes are found to take part only in the degeneration process for wide slot widths (e.g., $W_1 = 1.27$ mm). Furthermore, within the 18 modes calculated, no p-modes are found to take part in the degeneration process. Additionally, mode conversions from waveguide modes to c-modes are observed only for highly asymmetric structures.

The effect of different dielectric materials of the central layer, including $\varepsilon_r = 2.2$, 2.94, 6.15, and 10.5, on the existence of complex modes has also been investigated. Fig. 3(a) and (b) shows the results for $\varepsilon_r = 2.2$ and 10.5, respectively. When the relative dielectric constant is 2.2 (Fig. 3(a)), only the 4th and 5th waveguide modes are found to degenerate into complex modes in the range of 25–47.5 GHz. However, when the relative dielectric constant is increased to



(a)



(b)

Fig. 3. Dispersion characteristics of evanescent and complex modes in a shielded asymmetric CPW versus different central layer's permittivity. $W_1 = 0.508$ mm, $S = 0.508$ mm, $W_2 = 0.762$ mm, $G_1 = 1.016$ mm, $h_1 = h_3 = 3.429$ mm, $h_2 = 0.254$ mm, $\varepsilon_{r1} = \varepsilon_{r3} = 1$. (a) $\varepsilon_{r2} = 2.2$. (b) $\varepsilon_{r2} = 10.5$.

10.5 (Fig. 3(b)), waveguide modes 4 and 5 become complex for a wider range from 15–54.25 GHz. In addition, waveguide modes 8 and 9 are also found to convert into complex modes in the range of 15–67.78 GHz, and c-modes 6 and 7 degenerate into a complex mode pair over the 18.5–77.75 GHz range. For all of the cases considered so far, waveguide modes 4 and 5 are always found to degenerate into a complex mode pair. Moreover, lower-order c-modes are found to degenerate into complex modes only when using dielectric materials with a relatively high dielectric constant (e.g., $\varepsilon_r = 10.5$).

To demonstrate the effect of the central layer's thickness on the existence of the complex modes, we computed the eigenmodes' dispersions using 2.2-permittivity substrates with thicknesses of 0.127, 0.254, 0.508, and 0.7874 mm. The effect of using a thickness of 0.254 mm was described in Fig. 3(a), while the results for 0.7874 mm are presented in Fig. 4. It is apparent that when the thickness is increased from 0.254 to 0.7874 mm, waveguide modes 4 and 5 degenerate into complex modes within 5–53 GHz. Furthermore, c-modes 6 and 7 were also found to degenerate into complex modes from 4 to 74.5 GHz. Additionally, π-modes 8 and 9 were found to convert into complex modes for a short range of 9–11 GHz, then change back to evanescent modes. One of these p-modes undergoes a mode conversion and becomes a c-mode within the range of 35–45 GHz. This converted mode, once again, degenerates into the complex modes by combining with c-mode 6 at about 77 GHz, and remains so until 84.525 GHz,

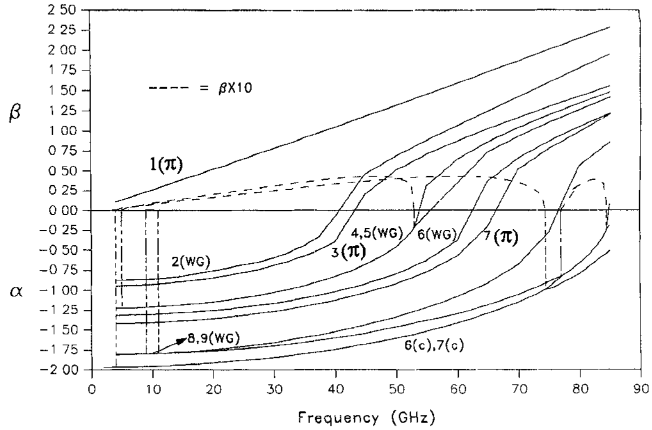


Fig. 4. Dispersion characteristics of evanescent and complex modes in the same CPW structure of Fig. 3(a) against thickness of the central layer. $h_2 = 0.7874$ mm.

when both modes change back again into evanescent c-modes. In all of the cases considered, lower-order waveguide modes are always found to degenerate into complex modes, except when the thickness is 0.127 mm. Furthermore, c-modes are found to convert into the complex pair only for relatively thick dielectric materials.

In order to investigate the numerical convergence of our calculated results, we also performed ample numerical tests. In all of the computed results, we have used three basis functions for each slot, which are sufficient for the considered structures for engineering purposes.

III. Conclusion

Calculations of possible complex modes existing in lossless shielded three-layer asymmetric CPW's have been presented based on the spectral domain method. The existence of the complex modes depends substantially on the asymmetry of the slots. These modes are found to exist at low microwave frequencies, even with a low-permittivity dielectric substrate. The waveguide modes are found to degenerate into complex modes more often than the c- or π-modes. We also found that lower-order c-modes degenerate into complex modes only for highly asymmetric structures and for thick or high-permittivity substrates. Additionally, conversions from waveguide to c-modes are also found to exist in highly asymmetric structures and when using dielectric materials of high permittivity or of large thicknesses. These findings suggest that, in order to properly design MIC's and MMIC's employing asymmetric CPW's, it is essential to determine all of the complex modes possibly existing in the asymmetric CPW's, even when the operating frequency is low or the dielectric substrate's permittivity is small.

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References

[1] C. P. Wen, "Coplanar waveguide: A surface strip transmission line suitable for nonreciprocal gyromagnetic device applications," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 1087–1090, Dec. 1969.

[2] A. K. Sharma and T. Itoh, Eds., "Special issue on modeling and design of coplanar monolithic microwave and millimeter-wave integrated circuits," *IEEE Trans. Microwave Theory Tech.*, vol. 41, Sept. 1993.

[3] T. Kitazawa and T. Itoh, "Asymmetrical coplanar waveguide with finite metallization thickness containing anisotropic media," *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 1427–1432, Aug. 1991.

[4] V. F. Hanna and D. Thebault, "Theoretical and experimental investigation of asymmetric coplanar waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. 32, pp. 1649–1651, Dec. 1984.

[5] P. J. B. Clarriocats and D. C. Taylor, "Evanescent and propagating modes of dielectric loaded waveguide," in *Proc. IEE*, Dec. 1964, vol. 111, no. 12, pp. 1951–1956.

[6] P. J. B. Clarriocats and K. R. Slinn, "Complex modes of propagation in dielectric loaded circular waveguide," *Electron Lett.*, pp. 140–146, 1965.

[7] V. A. Kalamyk, S. B. Rayeveskiy, and V. P. Ygvyumov, "An experimental verification of existence of complex waves in a two-layer, circular shielded waveguide," *Radio Eng. Electron Phys.*, pp. 16–19, 1978.

[8] A. S. Omar and K. F. Schunemann, "Formulation of the singular integral equation technique for planar transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 1313–1322, 1985.

[9] W. Huang and T. Itoh, "Complex modes in lossless shielded microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 163–165, Jan. 1988.

[10] C. C. Tzuang, J. T. Kuo, C. C. Tien, J. S. Jong, and T. H. Wang, "Complex modes in shielded planar microstripline," in *1989 IEEE MTTT-S Symp. Dig.*, pp. 495–497.

[11] T. Itoh and R. Mittra, "Spectral-domain approach for calculating the dispersion characteristic of microstrip line," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 498–499, July 1973.

[12] Y. Fujiki *et al.*, "Higher-order modes in coplanar type transmission lines," *Electron. Commun. Japan.* vol. 58-B, pp. 74–80, Feb. 1975.

[13] M. R. Lyons, J. P. K. Gilb, and C. A. Balanis, "Enhanced dominant mode operation of a shielded multilayer coplanar waveguide via substrate compensation," *IEEE Trans. Microwave Theory Tech.*, vol. 41, pp. 1564–1567, Sept. 1993.

Simple and Accurate Solutions of the Scattering Coefficients of E-Plane Junctions in Rectangular Waveguides

Anton Widarta, Shuzo Kuwano, and Kinchi Kokubun

***Abstract*—Simple and accurate solution of the scattering coefficients of the E-plane right-angle bend in rectangular waveguide is presented. The solution is obtained by the mode-matching method in which the electromagnetic fields in waveguides are matched with those in junction section formed by a sectoral region. In the same procedure, the solutions of the scattering coefficients of the E-plane T-junction and the cross junction can be also obtained easily. By using the numerical results, the scattering properties of the dominant modes and higher-order modes in the E-plane right-angle bend are examined in detail.**

I. Introduction

Rectangular waveguide junctions such as the right-angle bend, the T-junction and the cross junction are representatives of fundamental microwave circuits, which are applied to such as filters, multiplexers [1], and power dividers [2]. It is desirable that the scattering properties of these junctions are analyzed rigorously and that the obtained solutions are simple, convenient, and highly accurate over a wide frequency range.

The modeling of the above waveguide junctions is a canonical problem, and numbers of analysis methods have been proposed. Marcuvitz [3] represented the waveguide junction by an equivalent circuit. However, since the solution is an approximate one, there are limits in usable bandwidth and accuracy. In order to improve the equivalent circuits in [3], Lampariello and Oliner [4] presented new equivalent circuits for open and slit-coupled T-junctions. In [5], the T-junction boundary value problem is solved using equivalent circuit concept and leading to a calculation of the equivalent admittance matrix. The full numerical analyses, such as the finite-element method [6] and the boundary-element method [7], are useful techniques but require advanced computer processing.

On the other hand, the mode-matching method is a very efficient method for the analysis of these problems. However, since the electromagnetic fields in each junction section cannot be expanded in terms of the modal functions of a rectangular waveguide, some procedures are necessary in applying this method. Lewin [8] proposed a new expansion function in which the junction section of the right-angle bend is considered to be a region separated from the waveguides, though he did not derive the solution. In [9], the junction of the right-angle bend is divided into certain regions, and their boundaries are appropriately shorted. Then, the electromagnetic fields in the region are expanded in terms of the modal functions of a rectangular coordinate system. Similar strategies have been used to analyze the T-junction in [1], [2], and the hybrid junction in [10]. In the BCMM [11], instead of the point-matching [12], the contour-integral matching method in [13] is applied for the rigorous analysis of cascade arbitrarily shaped H-plane discontinuities in rectangular waveguides.

This paper presents a simple and accurate solution of the scattering coefficients of the E-plane right-angle bend in rectangular waveguide. The solution is obtained by the mode-matching method in which the electromagnetic fields in waveguide regions are matched with those in junction section formed by a sectoral region [14], [15] at the circumference boundary. This solution is expressed succinctly in form of matrix, and the formulations of the matrix elements are directly given. Hence, the solution is very simple and convenient. Since the solution is obtained by rigorous analysis, the obtained numerical results are highly accurate over a wide frequency range. In the same procedure, the solutions of the scattering coefficients of the E-plane T-junction and the cross junction can also be obtained easily. By using the numerical results, the scattering properties of the dominant modes and higher-order modes in the E-plane waveguide right-angle bend are examined in detail.

II. Theory

The rectangular waveguide I (region I) of width a and height b_1 and the rectangular waveguide II (region II) of width a and height b_2 are orthogonally joined to form an E-plane right-angle bend as shown in Fig. 1(a). Consider a TE_{10} mode is incident from region I on the junction, and LSE_{1n} $(n = 0, 1, 2, 3, \ldots)$ modes are scattered back into regions I and II. The field expansions are given as follows:

The incident wave is

$$H_x^{i\tau} = A^i \sin\left(\frac{\pi}{a}x_\tau\right)e^{j\beta^i z_\tau}. \quad (1)$$

The scattered wave is

$$H_x^\nu = \sum_{n=0}^{\infty} A_{1n}^\nu \sin\left(\frac{\pi}{a}x_\nu\right)\cos\left(\frac{n\pi}{b_\nu}y_\nu\right)e^{-j\beta_{1n}^\nu z_\nu} \quad (2)$$

where the indices τ and ν characterize the incidence region and

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